



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS:

Lecture 15: Pile Foundations under Lateral Loads: Conventional and Elastic Methods

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ERD of Foundations - Modules

1. Introduction
2. Shallow Foundations
- 3. Pile Foundations**
4. Well Foundations
5. SSI for Deep Foundations



Module - 3

Pile Foundations



Module -3: Chapters

1. Piles for Static Vertical Load
2. Lateral Loads on Piles
3. Dynamic Analysis of Piles
4. Design for Lateral and Dynamic Loads
5. Codal Provisions



Chapter 2: Lateral Loads on Piles

- **Introduction**
- **Conventional Method of Analysis**
- **Elastic Method using Winkler Approach**
- **Solution for Piles in Cohesionless Soil**
- **Solution for Piles in Clay**
- **Design Procedure for Piles**
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Introduction

Pile foundations are subjected to lateral load and moments which comes due to wind loads and seismic forces in buildings, earth pressure in retaining walls and abutments, water pressure in the water front structures. The conventional analysis assumes that the piles do not resist any load in the bending and the soil reaction to piles due to lateral movement is negligible.

Introduction (Continued...)

As per the conventional approach, vertical piles cannot resist lateral load, inclined (batters) piles are necessary to resist the lateral loads. This conventional approach is unrealistic and later theories which takes into account the bending of piles are more realistic for analysis of laterally loaded piles. The available approaches can be grouped into two categories.

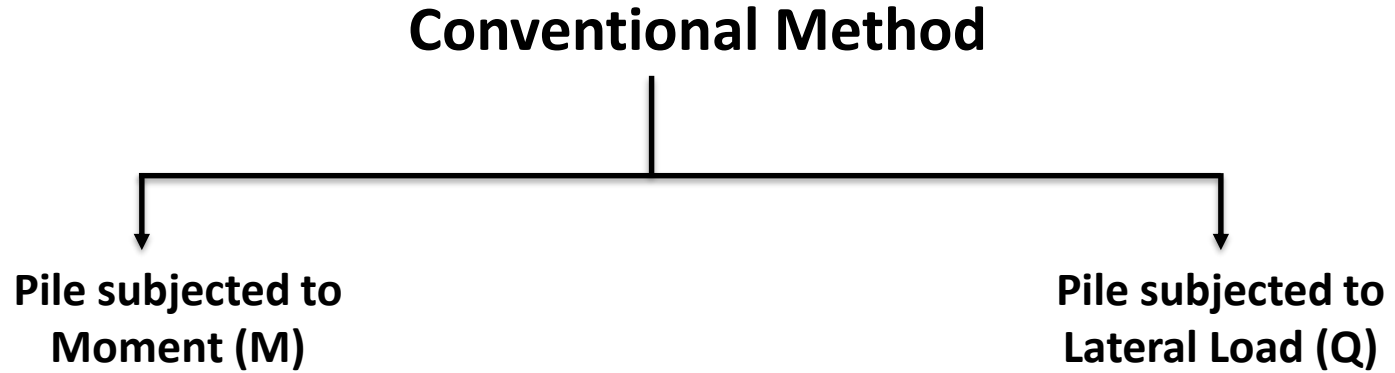
- Based on the elastic theories

- Based on the plastic theories

Introduction (Continued...)

Analysis based on the elastic theories (e.g. Reese and Matloch 1956) assume the soil to be replaced by a series of closely spaced, independent elastic springs (Winkler's assumptions), the equation for a beam on elastic foundation, utilizing the concept of subgrade modulus has been solved for different conditions and non-dimensional solutions have been given for a laterally loaded vertical pile to determine the variation of deflection, BM, shear, soil reaction etc. along the entire pile length. This requires knowledge of elastic constants namely E and μ .

Conventional Method



Pile Subjected to Moment (M)

➤ Assumptions

- i. The pile cap is rigid.
- ii. The load on each pile due to moment is proportional to its distance from the center of gravity of the pile group.
- iii. The settlement of a pile is proportional to the load acting on this pile.

Figure 1a shows a pile group subjected to a total vertical load $\sum V$ center of gravity of the group and a moment $\sum M$. Reactions in each pile caused by vertical load and moment are shown in Fig. 1b and 1c, respectively. In Fig. 1d, the loads are the sums of those in Fig. 1b and 1c. The same loads would be produced if $\sum V$ were to have an eccentricity e where

$$e = \frac{\sum M}{\sum V} \quad (1)$$

Pile Subjected to Moment (M) (Continued...)

From static equilibrium, $\sum M = 0$ Therefore,

$$\sum M = P_1 d_1 + P_2 d_2 + P_3 d_3 + P_4 d_4 \quad (2)$$

For the Linear distribution of the load,

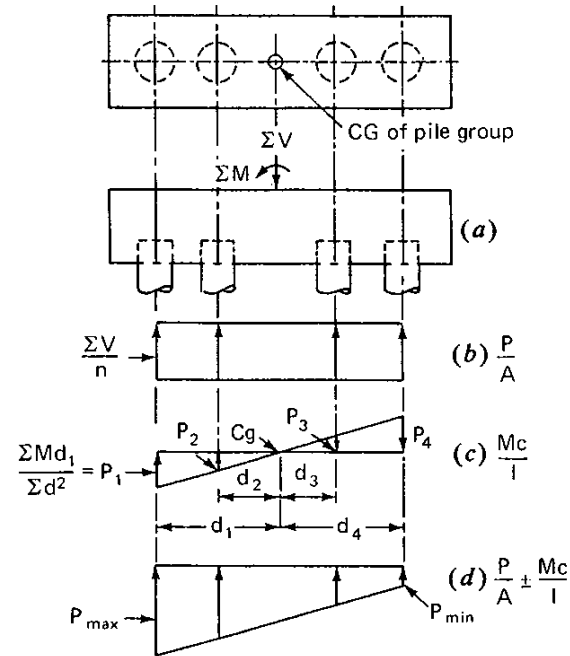
$$\frac{P_1}{d_1} = \frac{P_2}{d_2} = \frac{P_3}{d_3} = \frac{P_4}{d_4} \quad (3a)$$

or

$$P_2 = P_1 \frac{d_2}{d_1}, \quad P_3 = P_1 \frac{d_3}{d_1}, \quad P_4 = P_1 \frac{d_4}{d_1} \quad (3b)$$

Substitute the eqn. (3b) into (2)

$$\sum M = P_1 \left(\frac{d_1^2}{d_1} + \frac{d_2^2}{d_1} + \frac{d_3^2}{d_1} + \frac{d_4^2}{d_1} \right) \quad (4a)$$



Pile Subjected to Moment (M) (Continued...)

$$P_1 = \frac{\sum M d_1}{\sum d^2} \quad (4b)$$

$$P_n = \frac{\sum M}{\sum d^2} d_n \quad (4c)$$

Where, n stands for number of piles. If moment $\sum M'$ is acting in another plane.

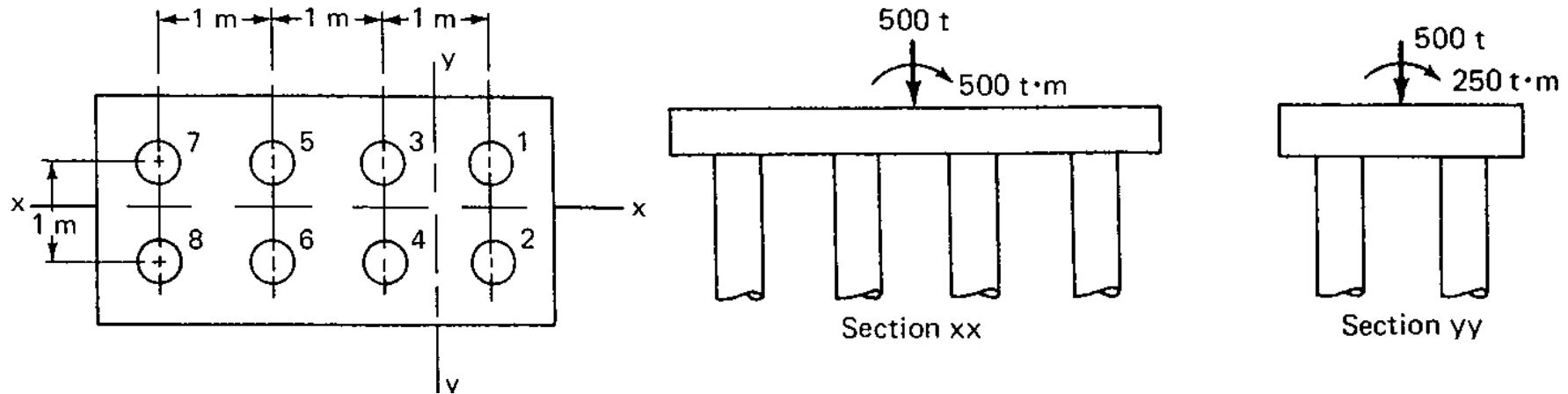
$$P'_m = \frac{\sum M'}{\sum d'^2} d'_m \quad (4d)$$

Where, m = number of piles in other plane and d'_m = distance of the pile from the center of gravity of the group. The total reaction force due to vertical load and moment is given by

$$P = \frac{\sum V}{n} \pm \frac{\sum M}{\sum d^2} d_n \pm \frac{\sum M'}{\sum d'^2} d'_m \quad (5)$$

Numerical Example- 1

The pile group shown in Fig is subject about the axes shown to a vertical load of 500t and a moment of 500 t-m and 250 t-m respectively. Calculate the maximum and minimum loads in the piles.



Numerical Example- 1 (Continued...)

Solution:

$$n = 8$$

$$\sum d_n^2 = 2(0.5^2 + 1.5^2)2 = 10 \text{ m}^2$$

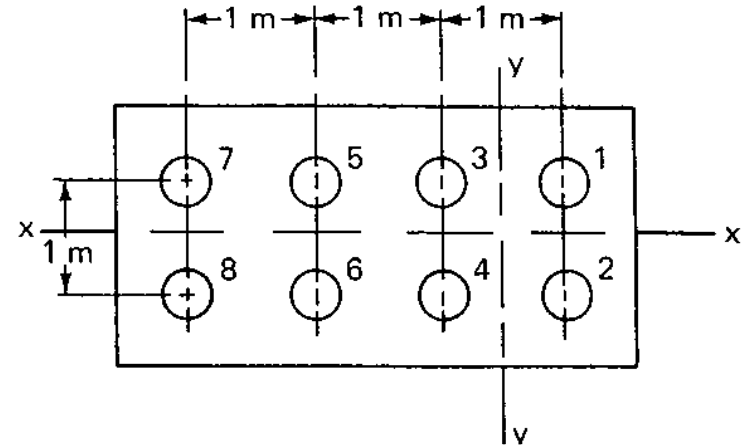
$$\sum d_m'^2 = 8(0.5^2)2 = 2 \text{ m}^2$$

Pile 1 is subjected to maximum load,

$$P_1 = \frac{500}{8} + \frac{500 \times 1.5}{10} + \frac{250 \times 0.5}{2} = 62.5 + 75 + 62.5 = 200 \text{ t}$$

Pile 8 is subjected to minimum load,

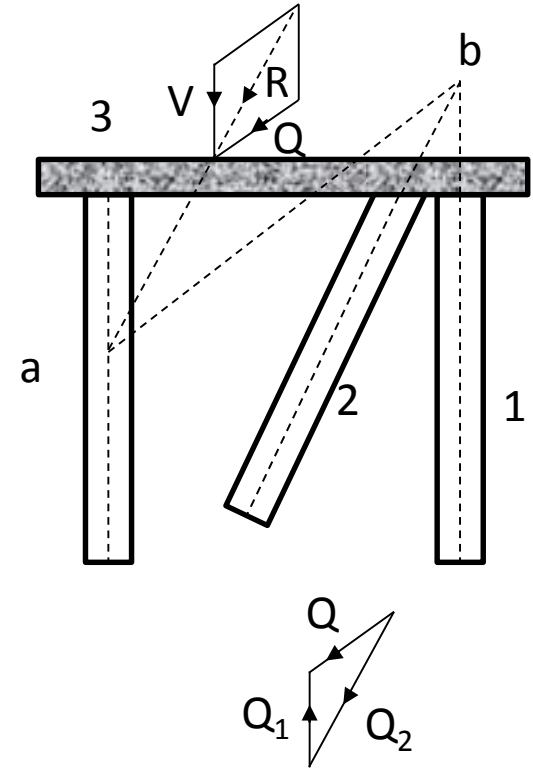
$$P_8 = \frac{500}{8} - \frac{500 \times 1.5}{10} - \frac{250 \times 0.5}{2} = 62.5 - 75 - 62.5 = -75 \text{ t}$$



Pile Subjected to Lateral Loads

Because the piles are assumed to withstand either tension or compression only, some of the piles must be vertically inclined so that the lateral load is properly carried by the pile group. According to Culmann's graphical method, piles are grouped according to their slopes. It is also assumed that the piles in each group are subjected to equal axial loads. The following steps are followed to analyze the pile groups:

1. Draw a dimensional sketch of the pile foundation
2. Locate the center line of each group of parallel piles.



File Subjected to Lateral Loads (Continued...)

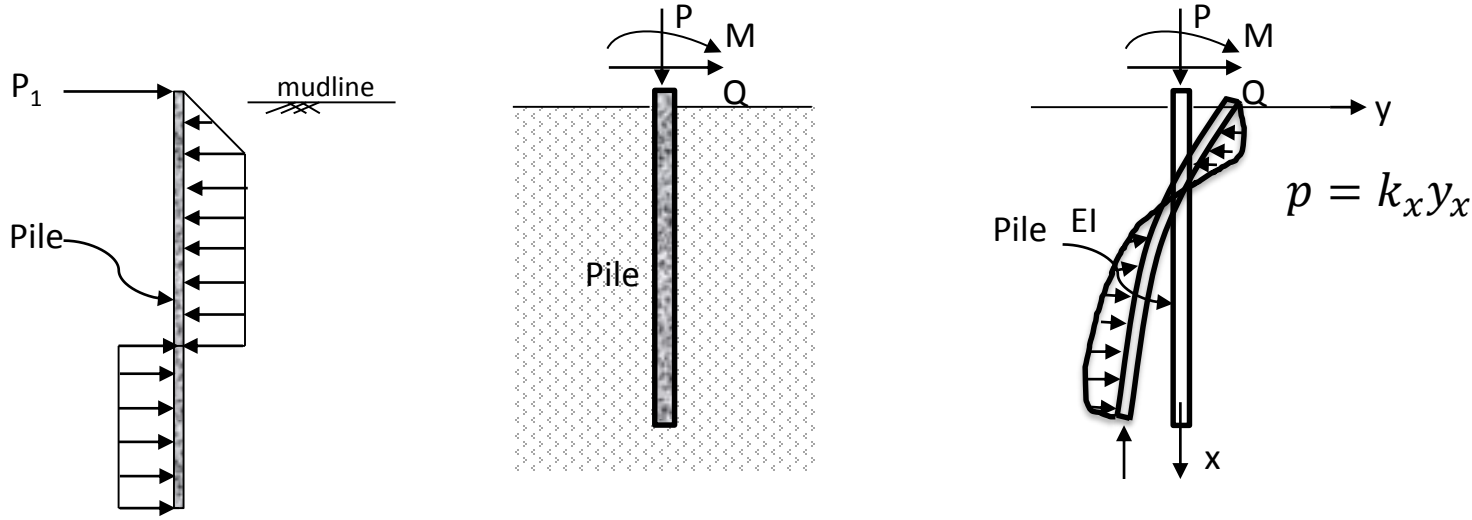
3. Draw the resultant R of all external forces applied on the pile foundation. R intersects the center line of the files in group 3 at point a .
4. Join ab where b is the point of intersection of the centerlines of groups 1 and 2.
5. Resolve R into Q and V . Q being parallel to ab and being vertical.
6. Group 3 is subjected to total axial force V .
7. Resolve load Q into axial loads along the centerlines of group 1 and group 2; Q_1 and Q_2 , act along the centerlines of groups 1 and 2, respectively.

If the piles are driven in more than three directions, the pile. reactions are compounded further. The analyses described above **ignore the capacity of soil** to withstand horizontal loads and the capacity of piles to withstand bending. Hence, these are not realistic analyses.



Limit Analysis

In limit analysis, it is assumed that maximum soil resistance acts against the pile as shown in Fig. In this analysis, it is assumed that the strength of the soil is constant with depth and that the pile deflects sufficiently to develop the full resistance



Elastic Method using Winkler Approach

A laterally loaded pile often been treated as a beam on an elastic foundation. According to Winkler (1867), the elastic soil medium is replaced by a series of infinitely close, independently, elastic spring. The stiffness of these spring may be written as

$$k = \frac{p}{y}$$

p = soil reaction = load / unit length (kN/m)

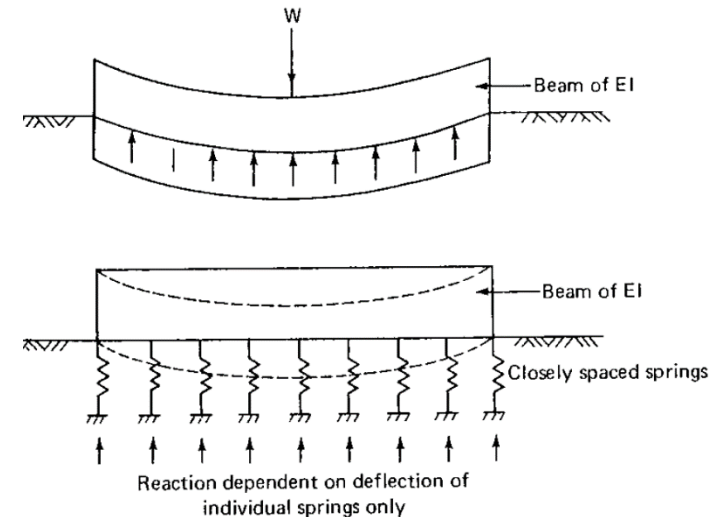
The stiffness k is also termed as the modulus of horizontal subgrade reaction.

The values of k varies with depth (x)

$$k_x = k_h \left(\frac{x}{L_s} \right)^n$$

n = coefficient = 1, <1, >1

k_h value of k at pile tip i.e. at $x = L_s$



Vertical Pile under the Lateral load (Continued...)

Terzaghi Recommendation

$n = 1$ for sand

$n = 0$ for clays

For sand,

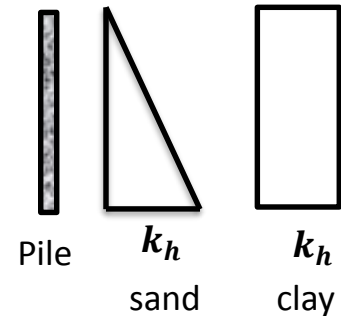
$$k_x = k_h \left(\frac{x}{L_s} \right) = \left(\frac{k_h}{L_s} \right) x$$

$$k_x = \eta_h x$$

$\eta_h = \frac{k_h}{L_s}$ = constant or coefficient of modulus
of subgrade reaction (kN/m^3)

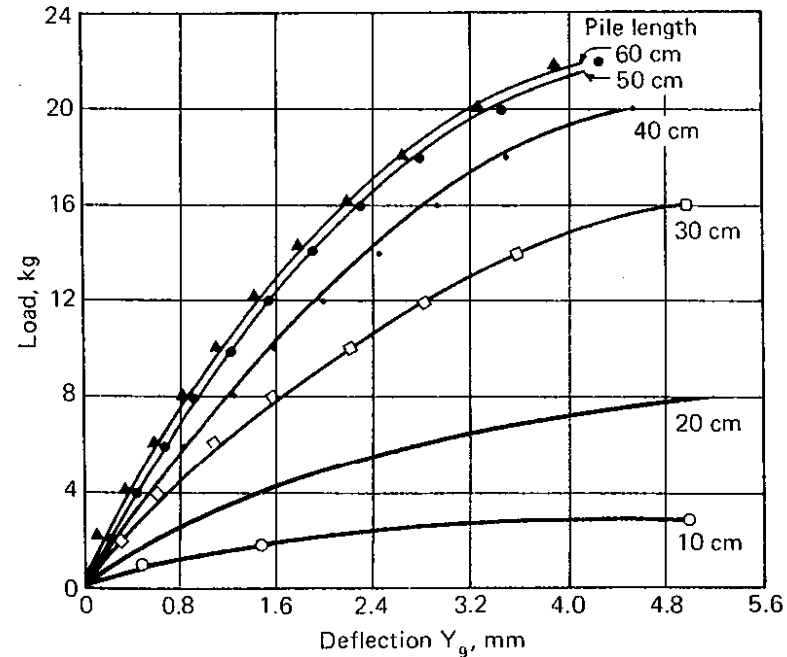
For clay,

$$k_x = k_h$$



Vertical Pile under the Lateral load (Continued...)

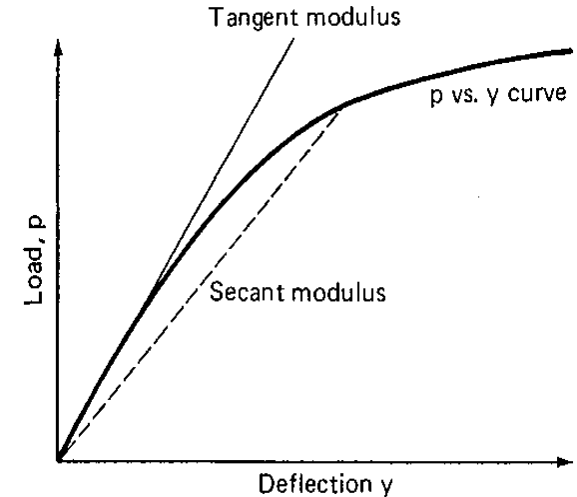
Prakash and (1967) reported test data on model piles subjected to horizontal load at 5 cm above the sand surface. The pile lengths were 10, 20, 30, 40, 50, and 60 cm. The deflections of piles at ground level, are shown in Fig. It is seen that as the pile length increases, the ground deflection of a pile of constant cross section decreases at the same load. This length L_α is defined as the infinite length of the pile. This would obviously be a function of pile stiffness EI and soil stiffness k .



Vertical Pile under the Lateral load (Continued...)

Stress, strain, and load deformation relations in soils are invariably nonlinear (Fig.). For loads of less than one-third to one-half their ultimate values, the load-deflection curve can be adequately expressed by a tangent modulus. For larger loads, a secant modulus is more appropriate. The secant modulus is dependent upon load level. Therefore, application of the theory of linear elasticity to the solution of pile problems is not strictly justified; however, to account for the nonlinearity between the load and pile deflection (or soil resistance and pile deflection), two approaches may be adopted.

1. One may employ repeated of the elastic theory.
2. The relationship between the secant modulus and the modulus can be defined in terms of strain level.



References

1. **Prakash S. (1981), "Soil Dynamics", McGraw-Hill Company, New York**
2. **Ranjan G. and Rao A.S.R. (2004), "Basic and Applied Soil Mechanics", New Age Int. Ltd., New Delhi.**

Thank You

